An Input-Output Approach to Valuing Non-Market Household Time

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Abstract

Unpaid household production is unmeasured, unvalued, and unseen in most economic policy studies. Family care work receives even less attention in economic policy and planning. However, unpaid household time and outputs are critical to the well-being of our economy. Historically, arguments against counting the economic contributions of household labor resulted from the difficulties of measuring and valuing non-market outputs. I demonstrate an economic model that combines unpaid family care activities and labor market participation within an Input-Output (I/O) framework to allow valuation of household care activities. Using the duality between time allocation and valuation, I determine implicit values of unpaid household production time in the same metric used in the I/O flows accounts, namely, the transactions-based, GDP denominated, monetary flows. This creates improved opportunities for economic assessments of policy impacts on both household and market labor.

Introduction

Parents use a combination of both paid (market) and unpaid household (non-market) care as they balance their roles as care givers, workers, and parents. However, economic analyses typically focus on market forms of care and exclude attention to non-market household production. This paper addresses that omission.

The failure of economic accounts to value household production is not limited to child care. U.S. National Income and Product Accounts (NIPA), on which

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estimates of Gross Domestic Product (GDP) are based, include no measures of non-market household production. The research reported in this paper bridges the gap between national accounts and non-market household production activities. This bridging is particularly important to understanding the linkage between child care and economic development.

This paper presents a method for valuing both market and non-market household activities in a comprehensive economic framework. It uses the economic value data from the NIPA, supplemented by physical data on time use from the American Time Use Survey (ATUS). First, I review the historical and conceptual bases used in measuring the economy. Next, I discuss the challenges of measuring the economic value of non-market household activities. I then review the concept of duality, which enables economists to infer value from information about production. In the Appendix, I present a simple numerical demonstration using mathematical optimization of input-output accounts with hypothetical time use constraints to compute "dual", or "shadow", values for non-market household time. The advantages of this implicit approach are that values are denominated in Gross Domestic Product (GDP) dollars and that no assignment of a wage rate to non-market time is required. I conclude with implications for policy of having measures that include both the market and non-market sectors of the economy.

Transactions and National Income Accounts

Partially in response to the uncertainty precipitated by the stock market crash of 1929 and the beginnings of the Great Depression, the U.S. Congress commissioned Simon Kuznets of Columbia University to construct guidelines and procedures for measuring aggregate economic activity in the U.S. In his transmittal letter to the Senate, Kuznets (1934) outlined the procedures that eventually became the foundation for producing the well-known measure of Gross National Product (GNP) and GDP. (Kuznets received a Nobel prize in economics in 1971.) Being an economist of the time, Kuznets relied on the principle of double-entry bookkeeping to suggest that 'market transactions', where there are buyers, sellers, and observable transactions prices, be the fundamental building blocks for the NIPA. This underlying 'market transactions principle' (Ruggles and Ruggles 1982, Reich 1991, 2001) continues to be codified into the widely accepted United Nations System of National Accounts (SNA) which is used by over 160 of the world's countries to do national accounting (United Nations Statistics Division 2004).

The transactions principle used in establishing national accounts leaves little room for including the value of commodities or services that occur outside of a market. Some transactions outside the market place are included in NIPA, such as services provided by government, as are some values that are not market transactions, such as the value of owner-occupied housing. The value of unpaid household production is, however, excluded. Since their inception in 1934, the national income accounts of the U.S. have omitted from their estimates of GDP any value for activities performed within households by nonfarm family members. 'Nonmarket' household production activities include such things as in-home meal preparation, laundry, house cleaning, and family care. Various estimates of the total increase of GDP value due to nonmarket production range from 12% to 80%, with most being in the 40%-60% range (Bryant et al. 1992, Hamdad 2003, Ironmonger 2001, Landefeld and McCulla 2000, Landefeld et.al 2005, Trewin 2000, Zick and Bryant 1983, 1990). Historically, the value of household production was omitted because it took place outside of a market and had no 'observable' transaction to be recorded, i.e. it is 'uncountable'. Additionally, consistent with the assumption that households are exclusively consumers rather than producers of economic goods and services, no estimates were made of the annualized value of durable goods used in household production, though there is the one exception of owner occupied houses.

Arguments against counting the economic contributions of household labor are deeply rooted in NIPA and this particular omission did not go unnoticed or unchallenged by household economists of the time (Reid 1934). Nor did the fact that, from the beginning, a major exception to the transactions principle was made by including in the national income accounts an imputed value for owner-occupied housing, also raising questions about the propriety of, or even the motivations for, omitting the majority of unpaid household production from the national accounts. There are sound theoretical reasons why unpaid non-market activities should not be included in the transactions-based national accounts (Reich 1991, 2001), but at the time of their inception, there was also an explicit determination that productive activities of housewives were not economic processes, i.e. 'they do not count'.[2] These national income accounting rules were presumed to be a set of coldly objective accounting principles. Today, it is recognized that the SNA is much more than a set of sterile rules. "National Accounts reflect underlying ideologies and paradigms. National Accounts construct realities, they do not simply represent them." (Cooper and Thompson 2000, p. 27)

Valuing Household Activities in Economic Analysis

"Economic theories have, for a long time, shown no interest in the productive function of the family. It has always been studied as a consumption unit." (Archambault 1987, p. 47) Bringing households into the mainstream of broader economic analysis is a relatively recent occurrence (Becker 1965, 1981 and Lancaster 1971). The areas of consumer choice and the work/leisure trade-off led the way. More recently, the articulation of a more general theory of household production has emerged. Obviously, households are the source of labor, an important factor of production for most market production. Unpaid household production arguably competes strongly with the visible, market transactions denominated, sectors of the economy, because many of these unpaid household production activities are time intensive.

Because the majority of household production was and continues to be provided by women, feminist economists (Folbre 1994, Himmelwait 1995) have led the effort to have this form of production 'counted', and have been joined by a wider range of consumer and household economists as well (Bryant et. al. 1992, Ironmonger 1989, 1996). "The recent contribution of the new 'home economics' school as well as of the feminist scholars is the recognition that production continues to take place in the home, as an aspect of consumption". (Silver 1987, p. 41) Based on research and dialogue, national income economists have suggested the creation of 'satellite' accounts for use in valuing non-market household production (Landefeld et. al. 2005, Trewin 2000). These accounts are 'based-on' the NIPA and allow for the formal imputation of a large and important component of national 'well being', while maintaining the transactional integrity of the national income accounts.

When imputing a value to non-market household time, the determination of the subset of possible activities to be considered is the first problem encountered. Most attempts have started with the presumption that only 'work' activities should be valued. Early on in the debate, Margaret Reid (Reid 1934) provided insight with her proposal of the 'third-person criterion', whereby she suggested that the distinction between unpaid work and non-work be whether or not a third person could be paid to do the unpaid activity in question. You could pay a third person to prepare a meal for you, but not to eat, and presumably enjoy, it for you. More recently, the definition of work, when used in a dichotomy of work/non-work, has been challenged (Himmelweit 1995). The 'personal relationships' and 'familial values' nature of caring labor, whether provided in the home or in the market (Folbre 2001), has been recognized, highlighting further difficulties in trying to value unpaid household productive activities involving care for family members or persons close to the caregiver. The method reported below could, but need not, distinguish between non-market work and non-work time, treating them equally or as restricted substitutes.

A second problem to overcome when imputing a value to non-market household time is deciding on the appropriate wage rate to apply to the unpaid time. There are several options usually discussed. A 'housekeeper' wage, where the prevailing housekeeper wage is used to value the time spent in all household production activities. A quality-adjusted 'replacement cost', where a specialist's wage for each household production activity is adjusted to reflect the average person's lower productivity compared to a professional. The 'opportunity cost' approach, which uses the average wage for all workers. Examples of studies using these methods appear in the literature (Bryant et. al.

1992, Hamdad 2003, Ironmonger 2001, Landefeld and McCulla 2000, Landefeld, et. al. 2005, Trewin 2000, Zick and Bryant 1983, 1990).

The need to value unpaid household time has persisted and is addressed in the recommendations of a recent National Academy of Sciences Panel to Study the Design of NonMarket Accounts (Abraham and Mackie 2005). "This argues for pursuing an approach that maintains a double-entry (input/output) structure; uses dollar values as a metric; seeks to value outputs at their marginal value (the market price) rather than their total value; and derives these marginal values from analogous, observable market transactions." (Abraham and Mackie 2005, p ES-2).

The approach I demonstrate below combines unpaid household time and labor market participation within an Input-Output (I/O) framework to allow for valuation of non-market household time. Using the duality between time allocation and valuation, an implicit rather than imputed marginal value for unpaid household time can be determined in the same metric used in the NIPA, namely, the transactions-based, GDP denominated, monetary flows.

Input/Output Analysis

Input-output analysis, as a theoretical framework and an applied economic tool, was developed by Wassily Leontief in his 1936 publication of the first input-output tables for the United States for the years 1919 and 1929. Since then, tables describing the interrelationships among various sectors of an economy have been constructed for over 90 countries. For the development of input-output methods and its application to important economic issues, Leontief was honored with a Nobel prize in Economic

Science in 1973. The integration of an input-output framework into the system of national accounts was developed and published in 1968 by the United Nations as a *System of National Accounts, Studies in Methods* (U. N. 2004). The integrated work earned Professor Richard Stone, a Nobel prize in Economic Science in 1984.

In the U.S., the national Input-Output (I/O) accounts, constructed by the Bureau of Economic Analysis (BEA), are a detailed form of the U.S. NIPA. By casting BEA's I/O tables in a mathematical programming framework and using household time use statistics as physical constraint (24 hours in a day) on the time use of persons, we can compute 'dual' (or 'shadow') values for household activities. These implicit values are denominated in the same transactions-based dollar denominated terms as GDP, are, by definition, marginal values, and are computed without the necessity of assigning any wage to non-market household time, thus satisfy all three of the National Academy Panel requirements (Abraham and Mackie p.15). A related approach was suggested by Gershuny (1987), whereby the monetarily enumerated national accounts are replaced by 'a time-based account' of economic structure which captures the chains of linked time use, much like I/O captures the chains of linked monetary flows. The method I demonstrate in the numerical appendix combines the monetary flows of I/O with time-based constraints determined from the ATUS.

The numerical appendix to this article presents a demonstration of the combination of I/O tables and mathematical programming, whereby a dual value can be determined for the use of time, both market and non-market, by households. This value is not an 'imputed' wage, but rather a marginal market output value. Like an I/O multiplier, it takes into account all the interindustry linkage effects of a reduction in paid labor time. While it is not an opportunity cost from an individual perspective, it is an opportunity cost from a total, economy-wide, market output perspective.

The Role of Duality

Duality is a concept that is found in many diverse disciplines including sociology (structure/agent), psychology (mind/body), economics (cost/technology), physics (wave/particle), and mathematics (primal/dual). The concept of duality simply refers to the possibility that there might be two or more, sometimes surprisingly different, ways for viewing the same phenomenon. These views are inextricably related and both may be needed to fully explain a single phenomenon, each view offering its own unique insights.

One of the important uses of duality in economics is the establishment of a formal connection between production technology and costs.[3] In mathematical optimization, the concept of duality is a highly developed relationship between an optimization problem, the 'primal', and its alternative, but equivalent, 'dual' problem. At the crossroad of economics and optimization, this means that a typical economic allocation problem, where a firm wishes to find an optimal *allocation* of its scarce resources to maximize some objective, such as returns or profit, has an equivalent economic *valuation* problem that optimizes the 'dual', or implicit, values of those scarce resources. These two views, *allocation* and *valuation*, of the same economic process are mathematically and economically equivalent. This relationship has been suggested as a method to price intra-firm transfers of goods and services (Eccles 1985). It is this particular dual relationship, between the optimal allocation of resources and the optimal values of these same resources, on which I build the analysis of unpaid household time.

Data on Market Transactions and Time Use

The data for the demonstration are taken from a study of a regional economy in Virginia. Like the NIPA developed by the Bureau of Economic Analysis, the regional economic tables measure the output of each sector in the regional economy and the sales and purchase linkages between each sector and households (Conner et al 1975). These provide the data on economic production relationships.

For data on time use, I use the ATUS. The ATUS is an on-going survey that was begun in 2003 and data are released annually. It samples an adult individual in families leaving the Current Population Survey and asks him/her how they spent their time the day previous to the interview. It differentiates paid work from household labor (cooking, cleaning etc.), leisure and sleep. The survey gives special attention to measuring child care at home. The ATUS demographic information includes, among its many variables, age, gender, the age distribution of children in the household, employment status, occupation, industry of employment, and marital status of the adult respondents (BLS 2007). Time use data, based on occupation and industry of employment, can be linked to the I/O industries in time use constraints to test for policy impacts on market and household care by different population subgroups.

Together these two data sources provide data on the market transactions based economic activity (NIPA) and on household time use (ATUS). By combining these in a constrained I/O model, I can include time specific constraints related to the activities of households in both paid and unpaid work and consumption in a mathematical programming formulation of I/O accounts.

Using the duality of allocation and valuation allows for the implicit valuation of household time in the same metric used in the I/O flows accounts, namely, the transactions based monetary flows in NIPA. These valuations can be interpreted as the transactions-based marginal value equivalents of unpaid, time intensive, household production activities. There are major advantages of this implicit value approach over the many imputed value approaches that have been suggested. No specific predetermination needs to be made about what household activities to consider as 'work' and no wage proxies need to be chosen. The historical NIPA series remain intact, yet the contribution of time to the transactions-based economy becomes visible and countable.

The demonstration, presented in the numerical appendix using regional economic data from four Virginia counties and data on time use from the ATUS, finds that in a constrained I/O model, the dual for non-market time is $11,173 (Appendix Table 7). This indicates that, at the margin, an increase in the use of one full-time equivalent unit of non-paid time has this impact on the total market GDP output level.

Implications for Future Research and Policy

The mathematical programming formulation can accommodate complicated interrelationships between time uses as well. For example, time use studies over the period 1965-2000 revealed that as mothers increased their time in market work, they reduced their time in housework, but not in childcare (Bianchi 2000, Bianchi 2006, Bianchi et al

2006). Similarly, additional education does not appear to alter the goods intensity of childcare, such that more educated parents do not reduce their time devoted to children as they increase their spending on children (Gronau and Hamermesh 2006).

Once the multifaceted connections between labor time/earnings and total, economy-wide market output value is made, a dual value for physical time units, denominated in GDP dollars, can be determined. The time constraints can be fully expressed by way of disaggregation of the industry sectors and detailed demographic information. The time-use relationships waiting to be discovered in the ATUS data can to be expressed in time use-industry-occupation-demographic relationships. This approach will enable economists and policymakers to better understand the connections between child care, or any non-market time uses, and economic development. It also satisfies the recommendations of the National Academy of Sciences (Abraham and Mackie 2005). It uses dollar values as a metric, it values outputs (time) at its marginal value (the shadow price), and it derives these marginal values from observable market transactions already included in NIPA. (Abraham and Mackie 2005, p ES-2).

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Appendix: A Numerical Demonstration

I/O tables of an economy have previously been formulated as mathematical optimization models, specifically as linear programs, (Brink and McCarl 1977). The demonstration example economy presented below is based on earlier work involving four Virginia counties.

If the economy is divided into *n* sectors, the fundamental I/O system can be represented as:[4]

1. $AX + Y = X$
2. $Y = X - AX$
3. $Y = (I - A)X$
4. $X = (I - A)^{-1}Y$

where:
X is an *n* x 1 vector of total market output
Y is an *n* x 1 vector of final demands
A is an *n* x *n* matrix of 'direct requirements',
AX is an *n* x 1 vector of intermediate demands, and
I is an *n* x *n* identity matrix

(I-A) is the *n* x *n* 'Leontief' matrix, and
$(I\text{-}A)^{-1}$ is the *n* x *n* 'Leontief' inverse

Equation 1 is the fundamental I/O equation, where the market output of each industry, X_i, is defined to be the sum of direct uses of industry i's market output in final demand, Y_i, and indirect uses in of its market output in intermediate production, $A_{i,j=1,n}$ X, by all the other industries. The solution to finding X in terms of Y involves computing the 'Leontief' matrix, equation 3, and the 'Leontief inverse', equation 4. The column sums of the 'Leontief inverse' are the familiar I/O output multipliers, which indicate the magnitude of change in total market output associated with a one unit change in final demand for a single industry.

For purely demonstration purposes, Appendix Table 1 presents the customary representation of an aggregated I/O transactions matrix, from a 1972 study, embodying the relationships from Equation 1 (Conner et al 1975).

Appendix Table 1. Demonstration I/O Transactions Table[5] ($100,000)

AX Y X

	Ag	Man	Trans	Whls Retail	Serv	Hhs	Inv	Gov	Exp	Total Sales
Inputs										
Agriculture	34	290	0	0	0	7	0	1	137	469
Manufacturing	25	1134	5	13	188	607	27	10	12303	14312
Transportation	6	304	54	25	80	22	5	3	111	610
Whls&Retail	13	490	18	45	156	1171	29	11	723	2656
Services	35	472	53	258	418	1387	573	229	816	4241
Households	208	3242	252	881	1816	869	0	244	1203	8715
Imports	77	5712	83	456	892	2539				
Depreciation	24	2157	129	805	446	489				
Government	47	511	16	173	245	1624				
Total Purchases	469	14312	610	2656	4241	8715				22288

In this table, inter-industry sales are read across the rows and inter-industry purchases down the columns. Intra-industry sales and purchases are represented on the diagonal. The first five rows and columns in Appendix Table 1 represent the 'industry', or 'processing', sectors. The next four columns represent the final demands, Y, and the next four rows represent non-industry payments sectors. The final row and column are total market output for each industry. Final demand, the sum of columns six through nine, C+I+G+E, is closely related to GDP.

The first step in determining the Leontief inverse is to compute the 'technical coefficients' or 'direct requirements', A, in Equation 2. This is done by dividing each element in the transactions matrix, Appendix Table 1, by its corresponding column total. This step presumes that there is a one-to-one relationship between the industries in the table and the commodities that these industries produce (ten Raa 2005). Appendix Table 2 contains the results of this division and is the A matrix used in Equations 1-4.

Appendix Table 2. Technical Coefficients

	Ag	Man	Trans	Whls Retail	Serv
Agriculture	**0.072**	**0.02**	**0**	**0**	**0**
Manufacturing	**0.053**	**0.079**	**0.0082**	**0.005**	**0.044**
Transportation	**0.013**	**0.021**	**0.08852**	**0.009**	**0.019**
Wholesale&Retail	**0.028**	**0.034**	**0.02951**	**0.017**	**0.037**
Services	**0.075**	**0.033**	**0.08689**	**0.097**	**0.099**
Households	**0.443**	**0.227**	**0.41311**	**0.332**	**0.428**
Imports	**0.164**	**0.399**	**0.13607**	**0.172**	**0.21**
Depreciation	**0.051**	**0.151**	**0.21148**	**0.303**	**0.105**
Government	**0.1**	**0.036**	**0.02623**	**0.065**	**0.058**
Total Purchases	1	1	1	1	1

Each coefficient in Appendix Table 2 represents the proportion of input purchases necessary from a row sector in order for the column sector to produce one dollar of market output, including purchases from itself. By construction, these column coefficients sum to one.

The Leontief matrix (I-A) is computed by subtracting the technical coefficients from an identity. This subtraction is done only for the industry rows and columns, because no technical relationships are posited between the industry, the payments, and the final demand sectors. While this step is sometimes described only as a intermediate step and is often omitted from introductory I/O literature, for our purposes, a deeper understanding of what the Leontief matrix represents is useful. While the technical coefficients in Appendix Table 2 represent the gross requirements per unit of market output for a column sector, the Leontief coefficients in Appendix Table 3 represent the net results of producing a unit of market output for a column sector.

Appendix Table 3. Leontief Matrix

	Ag	Man	Trans	Whls &Retail	Serv
Agriculture	**0.928**	**-0.02**	**0**	**0**	**0**
Manufacturing	**-0.053**	**0.9208**	**-0.008**	**-0.0049**	**-0.044**
Transportation	**-0.013**	**-0.021**	**0.911**	**-0.0094**	**-0.019**
Wholesale&Retail	**-0.028**	**-0.034**	**-0.03**	**0.9831**	**-0.037**
Services	**-0.075**	**-0.033**	**-0.087**	**-0.0971**	**0.901**

Because the rows and columns represent identical economic sectors, to capture intra-industry relationships, the diagonal elements must embody the net relationship of an industry's use of its own market output. For example, Agriculture requires 7.2¢ of its own market output to produce one dollar of market output (Appendix Table 2). Therefore, if Agriculture produced one dollar of market output, the net result would be only 92.8¢ of market output (Appendix Table 3). If opposite signs in the Leontief matrix are interpreted as indicating either uses or sources of market output, the negative values off the diagonal indicate a net use of an market output and the positive value on the diagonal indicates a net source of market output.[6]

The Leontief inverse, or 'interdependency coefficient' matrix, is shown in Appendix Table 4.

Appendix Table 4. Leontief Inverse

	Ag	Man	Trans	Whls &Retail	Serv
Agriculture	**1.079**	**0.02382**	**0.0003**	**0.0002**	**0.0012**
Manufacturing	**0.068**	**1.09025**	**0.0153**	**0.0109**	**0.0544**
Transportation	**0.019**	**0.02718**	**1.1002**	**0.0131**	**0.0249**
Wholesale&Retail	**0.037**	**0.04129**	**0.0377**	**1.0222**	**0.0445**
Services	**0.098**	**0.04893**	**0.1118**	**0.1118**	**1.1186**
Total	**1.301**	**1.23146**	**1.2654**	**1.1584**	**1.2436**

Elements in this table indicate the total market output requirement from a row sector that is needed for the column sector to produce enough market output to serve one dollar of final demand. Column totals from Appendix Table 4 indicate the economy-wide market output requirements needed to meet one dollar for final demand for that column sector and are referred to as the familiar 'output multipliers'. For example, for Agriculture to make a dollar of final demand available, it must produce \$1.079 in market output. This would include the direct needs for its own market output, and the indirect needs for Agricultural market output required by other sectors in order to supply Agriculture with their market output. Summed down all sectors, one dollar of final demand for Agricultural final demand requires \$1.30 in market output from all sectors. Similarly, the column totals for the other sectors indicate their output multipliers. When combined with auxiliary information, similar multipliers for employment and income can be computed.

<u>Integration of Input-Output (I/O) and Linear Programming (LP)</u>

The system of equations represented by Equation 4 has 'n' unknown variables, the sector market output levels, and 'n' equations. If there is a solution to this system, there will be only one solution. By definition, the observed economy that generated the flows table represents just such a solution. This system can be recast as a linear program, with an objective function, technical coefficients, right-hand-side constraints, and non-negativity of variables.[7]

$$\begin{array}{lrcl} \text{Maximize} & \Sigma X & & \\ \text{s.t.} & (I-A)\,X & = & Y \\ & X & \geq & 0 \end{array}$$

This system has an objective of maximized total market output, 'n' variables, X, and 'n' constraints, Y. The choice of objective is made so that the dual values are expressed in dollar values equivalent to GDP. Given that we are concerned with description of the system's marginal attributes, rather than predictions of larger changes, the choice of objective function determines the units of the dual values. Because the number of variables in this LP equals the number of constraints, it can have only one solution. As is more customary for LP formulations, we may change the flow equalities to less-than inequalities

$$\begin{array}{lrcl} \text{Maximize} & \Sigma X & & \\ \text{s.t.} & (I-A)\,X & \leq & Y \\ & X & \geq & 0 \end{array}$$

When any of these inequalities is satisfied at a strict inequality, the interpretation of that constraint is that final demand is not satisfied. In such a case, the Leontief matrix has no inverse and the system of equations cannot be solved. By incorporating a new set of 'artificial' variables that represents the actual levels of final demand that are met when the system is solved, Y_{actual}, and a new set of ‘n’ inequality constraints that require the actual final demand to be less than the originally observed demands, the extended LP formulation now has 3n variables, X’s, Y’s, and slacks, associated with the inequality constraints, and 2n constraints.

$$\begin{array}{lrcl} \text{Maximize} & \Sigma X & & \\ \text{s.t.} & -(I-A)\,X + Y_{actual} & = & 0 \\ & Y_{actual} & \leq & Y \\ & X, Y_{actual} & \geq & 0 \end{array}$$

Appendix Table 5 shows the LP formulation for the example along with the optimal primal solution values, the optimal X and Y_{actual} values, shown in the Solution Values row and the optimal dual (sometimes called shadow prices) for each inequality constraint shown in the Dual Values column.

Appendix Table 5. LP Formulation and Solution

	Ag	Man	Trans	Whls &Retail	Serv								Dual Values
Solution	469	14312	610	2656	4241	145	12947	141	1934	3005		22288	
Ag	-0.928	0.0203	0	0	0	1					=	0	
Man	0.0533	-0.9208	0.008	0.004895	0.0443		1				=	0	
Trans	0.0128	0.0212	-0.911	0.009413	0.0189			1			=	0	
Whls/Retail	0.0277	0.0342	0.03	-0.98306	0.0368				1		=	0	
Services	0.0746	0.033	0.087	0.097139	-0.901					1	=	0	
						1					<=	145	1.30105
							1				<=	12947	1.23146
								1			<=	141	1.26424
									1		<=	1934	1.15836
										1	<=	3005	1.24362

The LP solution values for X from Appendix Table 5 are identical to the market output values, X, from Appendix Table 1. The dual values give the change in the objective value, total market output (Σ X), associated with a one unit change in a right-hand-side, final demand, Y, in this case.

The dual values associated with the final demand constraints for the optimal LP solution are, within rounding, identical to the output multipliers reported in the column sums from the Leontief inverse in Appendix Table 4. Given the definition of dual values and the definition of the multipliers, this is not surprising. Both give the expected, final demand induced, change in total market output resulting from a unit change in the level of final demand.

Given that the optimal LP solution values for X are the same as the I/O values and the optimal dual values are the same as the I/O output multipliers, what is the advantage of using the LP formulation? Once the basic I/O problem is formulated as an LP, additional constraints and variables can be added to the basic I/O structure. This additional information need not be in the same GDP dollar units as the constraints in the original I/O problem. For our purposes, these additional constraints would involve the physical units of time use and availability for individuals in the I/O economy. This approach is similar to one suggested by Gershuny (1987). It captures the 'chain of provision' of time use for an economy through the interindustry relationships in the I/O. Rather than attempt to measure the direct and indirect time uses, the I/O-LP structure allows the direct and indirect time use chain to be embodied in the direct and indirect monetary flows in the I/O and in the direct and indirect time use relationships embodied in the LP time constraints.

Appendix Table 6 shows the formulation and optimal solution for an extended LP formulation where additional constraints on labor time are added to the original I/O constraints.

Appendix Table 6. Extended LP Formulation with Disaggregated Time Use

	Ag	Man	Trans	Whls /Ret	Serv						TIME	Other	Paid Care			Dual Values
Solution	469	14312	610	2656	4241	145	12947	141	1934	3005	84127	444669	17482		22288	
Ag	-0.9	0.02	0	0	0	1								=	0	
Man	0.05	-0.92	0.01	0.005	0.04		1							=	0	
Trans	0.01	0.021	-0.91	0.009	0.02			1						=	0	
Whls/Retail	0.03	0.034	0.03	-0.983	0.04				1					=	0	
Services	0.07	0.033	0.09	0.097	-0.9					1				=	0	
						1								<=	145	1.301051
							1							<=	12947	1.231463
								1						<=	141	1.264244
									1					<=	1934	1.158357
										1				<=	3005	1.243620
Paid Time	9.7	2.2	3.8	4.3	8.1						-1			=	0	0.000000
Tot Time											1	1	1	<=	546278	0.000000
Other Time												1		>=	444669	0.000000
Care Time													1	>=	17481	0.000000

The first additional constraint contains hypothetical technical coefficients describing the uses of labor, in full-time labor equivalents, needed per $100,000 of market output for each aggregate I/O sector. The American Time Use Survey (ATUS) measures how Americans spend their time in work, leisure, household production (including care work) and sleep. Summary statistics for 2005 (BLS 2006) indicate that the average person spends 15.4% of their time in paid work related activities, 3.2% of their time in non-paid care activities, and 81.4% of their time in other activities, which include sleeping, eating, and leisure. Using these summary statistics, the total time available in our example would be 546,276.7 FTE's. The FTE's in non-paid care would be 17,480.9 and the FTE's in other activities would be 444,669.2. The second added constraint requires that the sum of paid work time, non-paid time, and other activities time be no more than the total time available in the adult population. The third added constraint requires a minimum bound on sleep, personal, and leisure time. The fourth constraint puts a minimum bound on non-paid care time. For this demonstration, time is assumed to be directly substitutable between paid work time, care time, and other time. This treats all time as equal with respect to potential contributions to total GDP output, clearly an unrealistic case. More realistically, time constraints should be configured such that non-paid care time is a function of demographic characteristics, such as the number of children and adults needing care in the total population. The ATUS surveys will allow for investigation of more realistic time-use relationships.

For the optimal solution, a total of 84,126.6 full-time labor units are needed to produce the market output levels that allow the original final demand to be satisfied. With unconstrained time, the optimal solution to the extended LP problem in Appendix Table 6 is identical to the LP solution in Appendix Table 5.

Appendix Table 7 reports a paid time constrained solution where the lower bound on non-paid time is increased by one FTE, meaning that one more FTE of time must be devoted to non-paid time.

Appendix Table 7. Extended LP Formulation with Disaggregated Time Use and Binding Time Constraint

				Whls							T I M E		Paid			Dual
	Ag	Man	Trans	/Ret	Ser						Other	Care				Values
Solution values	469	14312	610	2656	4241	145	12947	141	1934	3005	84127	444669	17482		22288	
Agriculture	-0.928	0.02026	0	0	0	1								=	0	
Manufacturing	0.053	-0.9208	0.0082	0.0049	0.0443		1							=	0	
Transportation	0.013	0.02124	-0.91148	0.0094	0.0189			1						=	0	
Whls&Retail	0.028	0.03424	0.02951	-0.983	0.0368				1					=	0	
Services	0.075	0.03298	0.08689	0.0971	-0.901					1				=	0	
						1								<=	145	0.00000
							1							<=	12947	0.86199
								1						<=	141	0.67470
									1					<=	1934	0.55750
										1				<=	3005	0.18460
Paid Time	9.7	2.2	3.8	4.3	8.1						-1			=	0	0.11173
Total Time											1	1	1	<=	546278	0.11173
Other Time												1		>=	444669	-0.11173
Care Time													1	>=	17482	-0.11173

Given that the upper and lower bounds on population time use were computed to exhaust the adult population's total time, this would now become a binding constraint on time whereby not all of the economy's final demand can be met, necessitating the reduction of labor FTE's used in generating market output . In terms of the levels of sector market outputs and final demands that are met, a one unit increase in the need for care time has little discernable effect on the $100,000 units of market output reported in the table. However, the dual values for final demand and for the binding labor time now change discernibly. For Agriculture, which had an output multiplier/dual value of 1.30 in the unconstrained problem, the dual value is now zero. The optimal solution determines that it is Agricultural final demand which will go unmet as a result of the paid labor time shortage. Output multipliers for each of the other four processing sectors also decline appreciably as a result of the shortage of labor time. Labor time, which, by construction, had a zero dual value in the original I/O formulation (Appendix Table 6), now has a dual value of

$11,173 (Appendix Table 7), indicating that, at the margin, an increase in the use of one full-time equivalent unit of non-paid time has this impact on the total market output level. This is not an 'imputed' wage, but rather a marginal market output value. Like an I/O multiplier, it takes into account all the interindustry monetary and time-use linkage effects of the paid labor time. It also takes into account the direct and indirect time-use relationships embodied in the LP constraints. While the marginal value is not an opportunity cost from an individual perspective, it is an opportunity cost from a total, economy-wide, market output perspective.

Endnotes

[1] Dr. James Pratt is a Senior Research Associate in the Applied Economics and Management Department at Cornell University: contact jep3@cornell.edu. I would like to thank Dr. Mildred Warner of the Cornell University City and Regional Planning Department and David Kay of the Cornell Community and Rural Development Institute for their contributions to this research and helpful comments on this paper as well as the comments of the anonymous reviewers.

[2] "It may be doubted that the productive activities of housewives and other members of the family, rendered within the family circle, can be characterized as economic processes whose net product should be evaluated and included in national income." (Kuznets 1941, p.431)

[3] For example, it can be demonstrated that under certain assumptions, knowledge about a firm's production technology contains sufficient information to infer its cost and, by duality, given a firm's cost function, its production technology can be inferred (Shepard 1970).

[4] See Miernyk 1957 or Richardson 1972 for two of the many introductory presentations.

[5] An Economic Analysis for Development of the Counties, Cities, and Towns of the West Piedmont Planning District: An Economic Analysis of Interindustry Relationships. M.C . Conner, D. Pendse, and J. Pratt, Dept. of Ag. Econ., VPI&SU, 1975.

[6] In the mathematical Appendix section of this article, I will use the convention that positive coefficients represent 'uses' of resources and negative coefficients represent 'sources' of these resources, a 'negative' Leontief matrix.

[7] Family care experts correctly point-out that, "No linear input-output model will fully capture the complexities of child care. We should try to develop a better understanding of the nonlinearities, discontinuities, and surprises that are inherent in the production of human capabilities." (Folbre 2006, p. 50). While the demonstration example presented here is a linear program, more complicated 'nonlinearities' and 'discontinuities' in household time use that may be found in detailed analysis of the ATUS data could be easily represented in a nonlinear programming formulation of the same structure. The 'surprises' may be left until later.